DAMPING AND ENERGY DISSIPATION

Linear Viscous Damping
Is A Property Of The Computer Model
And Is Not A Property Of A Real Structure

19.1. INTRODUCTION

In structural engineering, viscous, velocity-dependent damping is very difficult to visualize for most real structural systems. Only a small number of structures have a finite number of damping elements where real viscous dynamic properties can be measured. In most cases modal damping ratios are used in the computer model to approximate unknown nonlinear energy dissipation within the structure.

Another form of damping, that is often used in the mathematical model for the simulation of the dynamic response of a structure, is proportional to the stiffness and mass of the structure. This is referred to as Rayleigh damping. Both modal and Rayleigh damping are used in order to avoid the need to form a damping matrix based on the physical properties of the real structure.

In recent years, the addition of energy dissipation devices to the structure has forced the structural engineer to treat the energy dissipation in a more exact manner. In this chapter, the limitations of modal and Rayleigh damping will be discussed. In addition, detailed algorithms will be presented for numerical solution, by iteration, for several different types of nonlinear energy dissipation devices.

19.2. DAMPING IN REAL STRUCTURES

It is possible to estimate an "effective" viscous damping ratio directly from laboratory or field tests of structures. One method is to apply a static displacement by attaching a cable to the structure and then suddenly removing the load by cutting the cable. If the structure can be approximated by a single degree of freedom, the displacement response will be of the form shown in Figure 19.1. For multi-degree-of-freedom structural systems, the response will involve the response of more modes and the test and the analysis method required to predict the damping ratios will be more complex.

It should be pointed out that the decay of the typical displacement response only indicates that energy dissipation is taking place. The cause of the energy dissipation may be due to many different effects such as material damping, joint friction and radiation damping at the supports. However, if it is assumed that all energy dissipation is due to linear viscous damping, the free vibration response is given by the following equation:

$$u(t) = u(0)\ e^{-\xi\omega t}\ \cos(\omega_D t) \tag{19.1}$$

where $\omega_D = \omega\sqrt{1-\xi^2}$

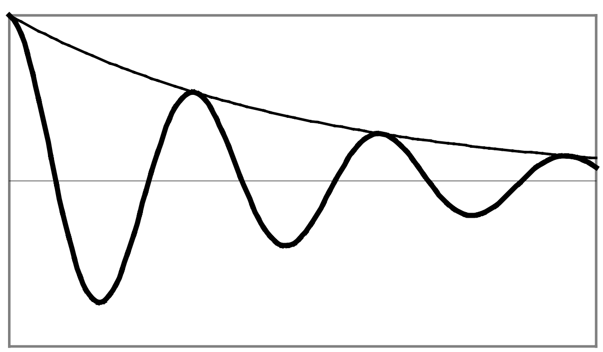

Figure 19.1. Free Vibration Test of Real Structures, Response vs. Time

Equation (19.1) can be evaluated at any two maximum points m cycles apart and the following two equations are produced:

$$u(2\pi n) = u_n = u(0)\, e^{-\xi\omega 2\pi n/\omega_D} \tag{19.2}$$

$$u(2\pi(n+m)) = u_{n+1} = u(0)\, e^{-\xi\omega 2\pi(n+m)/\omega_D} \tag{19.3}$$

The ratio of these two equations is

$$\frac{u_n}{u_{n+1}} = e^{\frac{2\pi\, m\xi}{\sqrt{1-\xi^2}}} \tag{19.4}$$

Taking the natural logarithm of this ratio and rewriting produces the following equation:

$$\xi = \frac{\ln(u_n / u_{n+m})}{2\pi m}\sqrt{1-\xi^2} \quad \text{or,} \quad \xi_{(i)} = \xi_0 \sqrt{1-\xi^2_{(i-1)}} \tag{19.5}$$

This equation can easily be solved for the damping ratio ξ by iteration. For example, if the decay ratio u_{n+m} is equal to 1.25 between two adjacent maximums, three iterations yield

$$\xi \approx 0.07023 \approx 0.07005 = 0.07005$$

Field testing of real structures, subjected to small displacements, indicates typical damping ratios are less than two percent. Also, for most structures, the damping is not linear and is not proportional to the velocity.

19.3. USE OF VISCOUS DAMPING IN ANALYSIS

In the elastic dynamic analysis of most structures subjected to earthquake motions it is very common to use five percent damping for all modes. However, this value, in most cases, has very little experimental or theoretical justification. Also, for multi degree-of-freedom systems, the use of modal damping violates dynamic equilibrium and the fundamental laws of physics. For example, it is possible to calculate the reactions, as a function of time, at the base of a structure by the following two methods:

First, the inertia forces, at each mass point, can be calculated in a specific direction by the multiplication of the absolute acceleration in that direction times the mass at the point. In the case of earthquake loading, the sum of all these forces must be equal to the sum of the base reaction forces in that direction since no other forces act on the structure.

Second, the member forces, at the ends of all members attached to reaction points, can be calculated as a function of time. The sum of the components of the member forces in the direction of the load is the base reaction force experienced by the structure.

In the case of zero modal damping these reaction forces, as a function of time, are identical. However, for nonzero modal damping, these reaction forces are significantly different. These differences indicate that linear modal damping introduces external loads, acting on the structure above the base, which are physically impossible. This is clearly an area where the standard "state of the art" assumption of modal damping needs to be re-examined and an alternative approach must be developed.

Energy dissipation exists in real structures. However, it must be in the form of equal and opposite forces between points within the structure. Therefore, a viscous damper, or any other type of energy dissipating device, connected between two points within the structure is physically possible and will not cause an error in the reaction forces. There must be zero base shear for all internal energy dissipation forces.

Another type of energy dissipation that exists in real structures is radiation damping at the supports of the structure. The vibration of the structure strains the foundation material near the supports and causes stress waves to radiate into the infinite foundation. This can be significant if the foundation material is soft relative to the stiffness of the structure. A spring, damper and mass at each support often approximate this type of damping.

19.4. NUMERICAL EXAMPLE

In order to illustrate the errors involved in the use of modal damping a simple seven-story building was subjected to a typical earthquake motion. Table 19.1 indicates the values of base shear calculated from the external inertia forces, which satisfy dynamic equilibrium, and the base shear calculated from the exact summation of the shears at the base of the three columns at each time increment.

It is of interest to note that the maximum values of base shear, calculated from two different methods, are significantly different for the same computer run. The only logical explanation is the existence of external damping forces that exist only in the mathematical model of the structure. Since this is physically impossible, the use of standard modal damping can produce a small error in the analysis.

TABLE 19.1. Comparison of Base Shear for Seven Story Building

Damping Percent	Dynamic Equilibrium BASE SHEAR (kips)	Sum of Column SHEARS (kips)	ERROR Percent
0	370.7 @ 5.355 Sec.	370.7 @ 5.355 Sec.	0.0
2	314.7 @ 4.690 Sec	318.6 @ 4.695 Sec	+1.2
5	253.7 @ 4.675 Sec	259.6 @ 4.690 Sec	+2.3
10	214.9 @ 3.745 Sec	195.4 @ 4.035 Sec	-9.1
20	182.3 @ 3.055 Sec	148.7 @ 3.365 Sec	-18.4

It is of interest to note that the use of only five percent damping reduces the base shear from 371 kips to 254 kips for this example. Since the measurement of damping in most real structures has been found to be less than two percent the selection of five percent reduces the results significantly.

19.5. STRUCTURES WITH LINEAR VISCOUS DAMPERS

It is possible to model structural systems with linear viscous dampers at arbitrary locations within a structural system. The exact solution involves the calculation of complex eigenvalues and eigenvectors and a large amount of computational effort. Since the basic nature of energy dissipation is not clearly defined in real structures and viscous damping is often used to approximate nonlinear behavior, this increase in computational effort is not justified since we are not solving the real problem. A more efficient method to solve this problem is to move the damping force to the right hand side of the dynamic equilibrium equation and solve the problem as a nonlinear problem using the FNA method.

19.6. STIFFNESS AND MASS PROPORTIONAL DAMPING

A very common type of damping used in the nonlinear incremental analysis of structures is to assume that the damping matrix is proportional to the mass and stiffness matrices. Or,

$$\mathbf{C} = \eta\,\mathbf{M} + \delta\,\mathbf{K} \tag{19.6}$$

This type of damping is normally referred to as Rayleigh damping. In mode superposition analysis the damping matrix must have the following properties in order for the modal equations to be uncoupled:

$$2\omega_n \zeta_n = \phi_n^T \mathbf{C} \phi_n \tag{19.7}$$

Due to the orthogonality properties of the mass and stiffness matrices, this equation can be rewritten as

$$2\omega_n \zeta_n = \eta + \delta\,\omega_n^2 \qquad \text{or,}\ \zeta_n = \frac{1}{2\omega_n}\eta + \frac{\omega_n}{2}\delta \tag{19.8}$$

It is apparent that modal damping can be specified exactly at only two frequencies in order to solve for η and δ in the above equation. In addition, the assumption of mass proportional damping implies the existence of external supported dampers that are physically impossible for a base supported structure. The use of stiffness proportional damping has the effect of increasing the damping in the higher modes of the structure for which there is no physical justification and can result in significant errors for impact type of problems. Therefore, the use of Rayleigh type of damping is difficult to justify for most structures. However, it continues to be used by many computer programs in order to obtain results for numerically sensitive structural systems.

19.7. NONLINEAR ENERGY DISSIPATION

Most physical energy dissipation in real structures is in phase with the displacements and is a nonlinear function of the magnitude of the displacements. Nevertheless, it is common practice to approximate the nonlinear behavior with an "equivalent linear damping" and not conduct a nonlinear analysis. The major reason for this approximation is that all linear programs for mode superposition or response spectrum analysis can consider linear viscous damping in an exact manner. This approximation is no longer necessary if the structural engineer can identify where and how the energy is dissipated within the structural system. The FNA method provides an alternative to the use of equivalent linear viscous damping. In this section various nonlinear devices will be discussed and their iterative solution algorithm will be summarized.

Base isolators are one of the most common types of predefined nonlinear elements used in earthquake resistant designs. Mechanical dampers, friction devices and plastic hinges are other type of common nonlinear elements. In addition, gap elements are required to model contact between structural components and uplifting of structures. A special type of gap element, with the ability to crush and dissipate energy, is useful to model concrete and soil types of materials. Cables, that can take tension only and dissipate energy in yielding, are necessary to capture the behavior of many bridge type structures.

19.8. BILINEAR PLASTICITY ELEMENT

The general plasticity element can be used to model many different types of nonlinear material properties. The fundamental properties and behavior of the element are illustrated in the figure shown below:

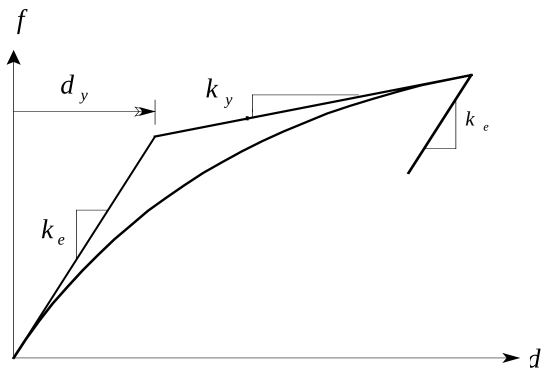

Figure 19.2. Fundamental Behavior of Bilinear Plasticity Element

Where k_e = initial linear stiffness

k_y = Yield stiffness

d_y = Yield deformation

The force-deformation relationship is calculated from

$$f = k_y d + (k_e - k_y) e \tag{19.9}$$

Where d is the total deformation and e is an elastic deformation term and has a range $\pm d_y$. It is calculated at each time step by the numerical integration of one of the following differential equations:

$$\text{If} \quad \dot{d}e \geq 0 \quad \dot{e} = (1 - |\frac{e}{d_y}|^n)\dot{d} \tag{19.10}$$

$$\text{If} \quad \dot{d}e < 0 \quad \dot{e} = \dot{d} \tag{19.11}$$

The following finite difference approximations, for each time step, can be made:

$$\dot{d} = \frac{d_t - d_{t-\Delta t}}{\Delta t} \qquad \text{And} \qquad \dot{e} = \frac{e_t - e_{t-\Delta t}}{\Delta t}$$

The numerical solution algorithm (six program statements) can be summarized at the end of each time increment Δt, at time "t" for iteration "i", in Table 19.2.

Table 19.2. Iterative Algorithm For Bilinear Plasticity Element

1. Change in deformation for time step Δt at time t for iteration i

$$v = d_t^{(i)} - d_{t-\Delta t}$$

2. Calculate elastic deformation for iteration i

$$\text{if } v e_t^{(i-1)} \le 0 \qquad e_t^{(i)} = e_{t-\Delta t} + v$$

$$\text{if } v e_t^{(i-1)} > 0 \qquad e_t^{(i)} = e_{t-\Delta t} + (1 - |\frac{e_{t-\Delta t}}{d_y}|^n)\, v$$

$$\text{if } e_t^{(i)} > d_y \qquad e_t^{(i)} = d_y$$

$$\text{if } e_t^{(i)} < -d_y \qquad e_t^{(i)} = -d_y$$

3. Calculate iterative force:

$$f_t^{(i)} = k_y d_t^{(i)} + (k_e - k_y) e_t^{(i)}$$

Note that the approximate term $\frac{e_{t-\Delta t}}{d_y}$ is used from the end of the last time increment rather than the iterative term $\frac{e_t^{(i)}}{d_y}$. This approximation eliminates all problems associated with convergence for large values of n. However, the approximation has insignificant effects on the numerical results for all values of n. For all practical purposes, a value of n equal to 20 produces true bilinear behavior.

19.9. DIFFERENT POSITIVE AND NEGATIVE PROPERTIES

The previously presented plasticity element can be generalized to have different positive, d_P, and negative, d_n, yield properties. This will allow the same element to model many different types of energy dissipation devices such as the double diagonal Pall friction element.

For constant friction the double diagonal Pall element has $k_e = 0$ and $n \approx 20$. For small forces both diagonals remain elastic, one in tension and one in compression. At some deformation, d_N, the compressive element may reach a maximum possible value. Friction slipping will start at the deformation u_P after which both the tension and compression forces will remain constant until the maximum displacement for the load cycle is obtained.

This element can be used to model bending hinges in beams or columns with non-symmetric sections. The numerical solution algorithm for the general bilinear plasticity element is given in Table 19.3.

Table 19.3. Iterative Algorithm For Non-Symmetric Bilinear Element

1. Change in deformation for time step Δt at time *t* for iteration *i*

   $$v = d_t^{(i)} - d_{t-\Delta t}$$

2. Calculate elastic deformation for iteration *i*

   if $v e_t^{(i-1)} \leq 0$ $\quad e_t^{(i)} = e_{t-\Delta t} + v$

   if $v e_t^{(i-1)} > 0$ $\quad$ and $e_{t-\Delta t} > 0$

   $$e_t^{(i)} = e_{t-\Delta t} + \left(1 - \left|\frac{e_{t-\Delta t}}{d_y}\right|^n\right) v$$

   if $v e_t^{(i-1)} > 0$ $\quad$ and $e_{t-\Delta t} < 0$

   $$e_t^{(i)} = e_{t-\Delta t} + \left(1 - \left|\frac{e_{t-\Delta t}}{d_n}\right|^n\right) v$$

   if $e_t^{(i)} > d_y$ $\quad e_t^{(i)} = d_y$

   if $e_t^{(i)} < -d_y$ $\quad e_t^{(i)} = -d_y$

3. Calculate iterative force at time t :

   $$f_t^{(i)} = k_y d_t^{(i)} + (k_e - k_y) e_t^{(i)}$$

19.10. THE BILINEAR TENSION-GAP-YIELD ELEMENT

The bilinear tension only element can be used to model cables connected to different parts of the structure. In the retrofit of bridges this type of element is often used at expansion joints to limit the relative movement during earthquake motions. The fundamental behavior of the element is summarized in Figure 19.3. The positive number d_0 is the initial pre-stress deformation. A negative number specifies the axial deformation associated with initial cable sag. The permanent element yield deformation is u_p.

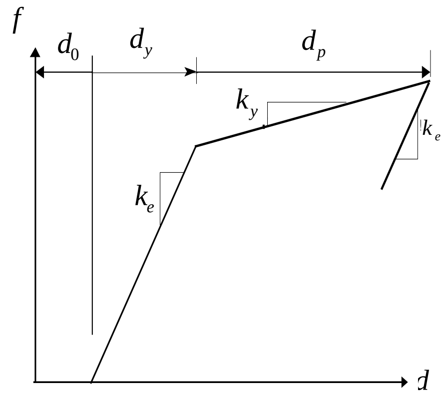

Figure 19.3. Tension-Gap-Yield Element

The numerical solution algorithm for this element is summarized in Table 19.4. Note that the permanent deformation calculation is based on the converged deformation at the end of the last time step. This avoids numerical solution problems.

Table 19.4. Iterative Algorithm -Tension-Gap-Yield Element

1. Update Tension Yield Deformation From Previous Converged Time Step

   $y = d_{t-\Delta t} + d_0 - d_y$

   if $y < d_p$ then $d_p = y$

2. Calculate Elastic Deformation for Iteration (*i*)

   $d = d_t^{(i)} + d_0$

   $e_t^{(i)} = d - d_p$

   if $e_t^{(i)} > d_y$ then $e_t^{(i)} = d_y$

3. Calculate Iterative Force:

   $f_t^{(i)} = k_y(d_t^{(i)} - d_0) + (k_e - k_y)e_t^{(i)}$

   if $f_i^{(i)} > 0$ then $f_i^{(i)} = 0$

19.11. NONLINEAR GAP-CRUSH ELEMENT

Perhaps the most common type of nonlinear behavior that occurs in real structural systems is the closing of a gap between different parts of the structure; or, the uplifting of the structure at its foundation. The element can be used at abutment-soil interfaces and for modeling soil-pile contact. The gap/crush element has the following physical properties:

1. The element cannot develop a force until the opening d_0 gap is closed
2. The element can only develop a compression force
3. The crush deformation d_p is always a monatomic decreasing negative number

The numerical algorithm for the gap-crush element is summarized in Table 19.5.

Table 19.5. Iterative Algorithm To Model Gap-Crush Element

1. Update Crush Deformation From Previous Converged Time Step

$$y = d_{t-\Delta t} + d_0 + d_y$$

if $y > d_n$ then $d_n = y$

2. Calculate Elastic Deformation:

$$e_t^{(i)} = d_t^{(i)} + d_n - d_y$$

if $e_t^{(i)} < -d_y$ then $e_t^{(i)} = -d_y$

3. Calculate Iterative Force:

$$f_t^{(i)} = k_y(d_t^{(i)} - d_0) + (k_e - k_y)e_t^{(i)}$$

if $f_i^{(i)} > 0$ then $f_i^{(i)} = 0$

The numerical convergence of the gap element can be very slow if a large elastic stiffness term k_e is used. The user must take great care in selecting a physically realistic number. In order to minimize numerical problems, the stiffness k_e should not be over 100 times the stiffness of the elements adjacent to the gap. The dynamic contact problem between real structural components often does not have a unique solution. Therefore, it is the responsibility of the design engineer to select materials at contact points and surfaces to have realistic material properties that can be predicted accurately.

19.12. VISCOUS DAMPING ELEMENTS

Linear velocity-dependent energy-dissipation forces exist in only a few special materials subjected to small displacements. In terms of equivalent model damping, experiments indicate that they are a small fraction of one percent. Manufactured mechanical dampers cannot be made with linear viscous properties since all fluids have finite compressibility and nonlinear behavior is present in all manmade devices. In the past it has been common practice to approximate the behavior of these viscous nonlinear elements by a simple linear viscous force. More recently, vendors of these devices have claimed that the damping forces are proportional to a power of the velocity. Experimental examination of a mechanical device indicates a far more complex behavior that cannot be represented by a simple one-element model.

The FNA method does not require that these damping devices be linearized or simplified in order to obtain a numerical solution. If the physical behavior is understood it is possible for an iterative solution algorithm to be developed which will accurately simulate the behavior of almost any type of damping device. In order to illustrate the procedure let us consider the device shown in Figure 19.4.

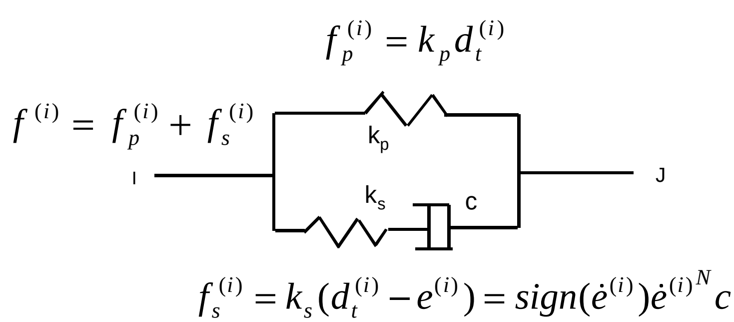

Figure 19.4. General Damping Element Connected Between Points I and J

It is apparent that the total deformation, $e_t^{(i)}$, across the damper must be accurately calculated in order to evaluate the equilibrium within the element at each time step. The finite difference equation used to estimate the damper deformation at time t is

$$e_t^{(i)} = e_{t-\Delta t} + \int_{t-\Delta t}^{t} \dot{e}_\tau^{(i)} d\tau = e_{t-\Delta t} + \frac{\Delta t}{2}(\dot{e}_{t-\Delta t} + \dot{e}_t^{(i)}) \tag{19.12}$$

A summary of the numerical algorithm is summarized in Table 19.6.

Table 19.6. Iterative Algorithm For Nonlinear Viscous Element

1. Estimate damper force from last iteration:
   $$f_c^{(i)} = k(d_t^{(i)} - e_t^{(i-1)})$$
2. Estimate damper velocity:
   $$\dot{e}_t^{(i)} = (\frac{f_c^{(i)}}{c})^{\frac{1}{N}} sign(f_c^{(i)})$$
3. Estimate damper deformation:
   $$e_t^{(i)} = e_{t-\Delta t} + \frac{1}{2\Delta t}(\dot{e}_{t-\Delta t)} + \dot{e}_t^{(i)})$$
4. Calculate total iterative force:
   $$f_t^{(i)} = k_p d_t^{(i)} + k_s(d_t^{(i)} - e_t^{(i)})$$

19.13. SUMMARY

The use of linear modal damping, as a percentage of critical damping, has been used to approximate the nonlinear behavior of structures. The energy dissipation in real structures is far more complicated and tends to be proportional to displacements rather than proportional to the velocity. The use of approximate “equivalent viscous damping” has little theoretical or experimental justification.

It is now possible to accurately simulate the behavior of structures with a finite number of discrete energy dissipation devices installed. The experimental determined properties of the devices can be directly incorporated into the computer model.